

# Real Exchange Rate Dynamics in the New-Keynesian Model

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## Abstract

This paper studies the real exchange rate adjustment process in the baseline small open economy New-Keynesian framework. The paper shows that i) the version of the model with real shocks replicates the persistence and the hump-shaped dynamics of the real exchange rate observed in data ii) the model cannot simultaneously match the observed dynamics of the real exchange rate and the close co-movement between the real and nominal currency returns. Thus, the baseline framework is not capable of fully capturing the real exchange rate adjustment process.

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*Keywords:* Real exchange rate adjustment, Nominal-real exchange rate co-movement, New Keynesian model, Monetary policy rule

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# 1. Introduction

Empirical evidence suggests that the real exchange rate is highly volatile and adjusts to shocks rather slowly. Moreover, a great number of studies show that the real exchange rate exhibits hump-shaped dynamics.<sup>1</sup> Steinsson (2008) argues that a two-country sticky price model with real shocks matches the real exchange rate dynamics. As discussed in Steinsson, the reason for the hump-shaped response of the real exchange rate in the baseline model is that, following a negative real shock, the ex-ante real interest rate decreases on impact but next becomes positive in the subsequent periods. Iversen and Söderström (2014), on the other hand, show that the findings in Steinsson (2008) heavily depend on the specification of the policy rule.

In the current paper, I confirm the results in Steinsson (2008) by showing that a baseline small open economy New-Keynesian model with real shocks replicates the behaviour of the real exchange rate. However, the model fails to capture the observed strong co-movement between the nominal and the real currency returns across the real exchange rate adjustment process.<sup>2</sup> To generate the required behaviour of the real exchange rate, the model assumes a policy rule with a sluggish response to current inflation. However, the latter implies a rather weak co-movement between the nominal and real currency returns.

A lot of papers have studied the ability of sticky price models to rationalize the volatility and persistence of real exchange rates. Chari et al. (2002) argue that such models can explain the volatility of the real exchange rate but cannot account for its persistence. Various attempts have been introduced to solve the latter issue by making modifications to the baseline setup: strategic complementarities in price setting, nominal wage rigidities, persistent monetary policy, etc.(see among others, Bergin and Feenstra (2001), Groen and Matsumoto (2004), Bouakez (2005) Benigno (2004), Engel (2012) and Carvalho and Nechio (2015)). These features increase the persistence of the real exchange rate. However, they are not

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<sup>1</sup>See among others, (Huizinga (1987), Eichenbaum and Evans (1995), Cheung and Lai (2000), Iversen and Söderström (2014), and Burstein and Gopinath (2014).

<sup>2</sup>See, among others, Mussa (1986), Finn (1999) and Monacelli (2004), Burstein and Gopinath (2014).

sufficient to explain the hump-shaped dynamics of the real exchange rate. Steinsson (2008) argues that an open economy sticky price model with real shocks replicates the behaviour of the real exchange rate. The current paper, on the contrary, shows that the baseline model cannot simultaneously match the observed dynamics of the real exchange rate and the close co-movement between the real and nominal currency returns.

The rest of the paper proceeds as follows. The second section describes the model with baseline parametrization. The third section outlines the main results. The fourth section looks deeper into the problem of co-movement between the real and nominal currency returns. The final section summarizes and concludes.

## 2. A Basic Small Open Economy NK Model

Given standard assumptions on the preferences and the production function, the equilibrium conditions of the model are given by:<sup>3</sup>

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{ht+1}) \quad (2.1)$$

$$\pi_{ht} = \beta E_t \pi_{ht+1} + \lambda \left( \frac{\phi + \psi}{1 - \psi} \phi + \sigma \right) y_t + u_t \quad (2.2)$$

$$\Delta q_t = (1 - \alpha) \sigma \Delta y_t \quad (2.3)$$

$$\Delta e_t = \frac{1}{1 - \alpha} \Delta q_t + \pi_{h,t} \quad (2.4)$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) (\Phi_\pi \pi_{ht} + \Phi_y y_t) \quad (2.5)$$

$$u_t = \rho_u u_{t-1} + e_{u,t} \quad (2.6)$$

(2.1) and (2.2) are the dynamic IS equation and the New Keynesian Phillips Curve, respectively. Monetary policy is conducted with a Taylor given by (2.5). Equation (2.3) describes the dynamics of the real currency return is derived from the international risk sharing condi-

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<sup>3</sup>See Gali and Monacelli (2005) for the derivations.

tion under complete financial markets. Equation (2.4) defines the nominal currency return.  $y_t$  is output,  $\pi_t$  is overall inflation rate and  $i_t$  is the nominal interest rate. Also,  $\pi_{h,t}$  denotes inflation rate for domestically produced goods,  $q_t$  and  $e_t$  denote real end nominal exchange rate, respectively. Finally,  $\Delta$  denotes first difference.  $u_t$  is a composite of different real shocks: productivity shocks, cost-push shocks, government consumption shocks, etc. It is assumed that  $u_t$  follows an AR(1) process. In the analysis, I do not make a distinction between real shocks as the latter impact the real exchange rate in a similar manner.<sup>4</sup>  $\beta$  is the discount factor,  $\alpha$  measures “openness” of the economy. Furthermore,  $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\psi}{1-\psi+\psi\epsilon}$ , where  $\theta$  denotes the amount of price stickiness,  $\psi$  measures curvature of the production function and  $\epsilon$  is the price elasticity of demand.  $\phi$  is the inverse of Frisch elasticity of labor supply.  $\sigma$  measures sensitivity of output to interest rate changes. In an open economy, it also depends on the degree of openness and the elasticity of substitution between imported and domestically produced goods.  $\Phi_\pi$  and  $\Phi_y$  are the response parameters to domestic inflation and output.

The baseline parametrization follows that of Steinsson (2008). I set  $\beta = 0.99$ ,  $\sigma = 5$  and  $\phi = 3$ .  $\alpha$  is set to 0.06. Prices remain fixed for 3 quarters on average, i.e.  $\lambda = 0.085$ . Furthermore, I set  $\epsilon = 10$  and  $\psi = 0.15$ . The slope of the Philips curve, thus, is  $\kappa = \lambda(\frac{\phi+\psi}{1-\psi}\phi + \sigma) = 0.27$ . The Taylor rule parameters are as follows:  $\Phi_\pi = 2$ ,  $\Phi_y = 0.5$  and  $\rho_i = 0.85$ . Table 1 summarizes the parameter values.

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<sup>4</sup>The proceeding analysis explains why other shocks (in particular, monetary shocks) are not considered in the model.

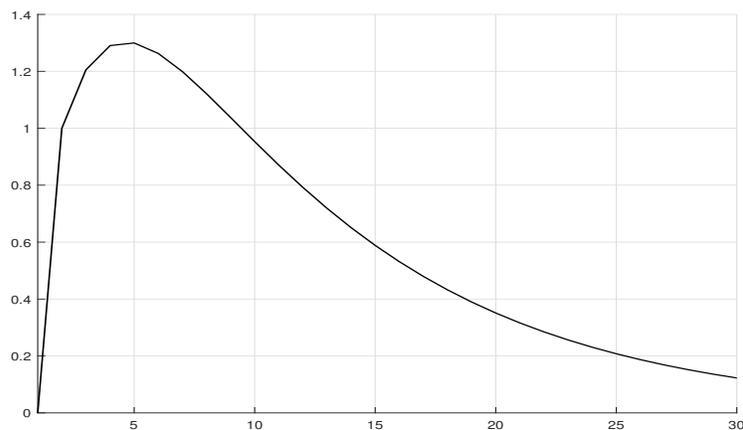
Table 1. Baseline Calibration

Parameter	Description	Value
$\beta$	Time discount factor	0.99
$\sigma$	Elasticity of intertemporal substitution	5
$\kappa$	Slope of Philips curve	0.27
$\alpha$	Country openness	0.04
$\rho_i$	Interest rate smoothing	0.85
$\Phi_\pi$	Inflation response	2
$\Phi_y$	Output response	0.5
$\rho_u$	Shock persistence	0.9

### 3. Model results

In the current section, I ask whether the model described above can replicate the empirical facts about the dynamics of the real exchange rate and the close correlation between the nominal and the real currency returns.

Figure 1 plots the normalized response of the real exchange rate to a positive supply shock. The impulse response function displays a pronounced hump peaking at about 1.3



Notes: The impact response of the real exchange rate is normalized to unity.

**Figure 1. Real exchange rate response to an increase in supply shock**

before it starts dying out. The real exchange rate does not fall below 1 until 9 quarters after the shock. Table 2 reports that the  $\frac{UL}{HL}$ , a key measure of the degree of hump in the impulse response, is 0.50 (row “Baseline”). That is, 50 percent of the time that it takes the real exchange rate to fall below  $\frac{1}{2}$ , it is above 1. Finally,  $HL$  is considerably larger than  $QL - HL$  which means that the response dies out faster from  $\frac{1}{2}$  to  $\frac{1}{4}$  than from 1 to  $\frac{1}{2}$ . This is also an indication of a high degree of hump in the response function. The reported values of  $HL$ ,  $2QL - HL$  and  $\frac{UL}{HL}$  under the baseline calibration are very much in line with the median estimates in Steinsson (2008).

To understand why the real exchange rate exhibits hump-shaped response to a supply shock, consider the UIP condition in the real form:

$$q_t = E_t q_{t+1} - r_t \quad (3.1)$$

where  $r_t = i_t - E_t \pi_{t+1}$  is the ex-ante real interest rate. Iterate forward the above difference equation to get:

$$q_t = E_t q_{t+j} - E_t \sum_{i=0}^j r_{t+i} \quad (3.2)$$

Take  $j \rightarrow \infty$ . The long run log real exchange rate is 0,  $\lim_{j \rightarrow \infty} E_t q_{t+j} = 0$ , therefore  $q_t$  is determined by the dynamics of ex ante real interest rates:

$$q_t = -E_t \sum_{i=0}^{\infty} r_{t+i} \quad (3.3)$$

If the real exchange rate is to be hump-shaped, the sum of the real interest rates should be hump-shaped. Thus, as a response to a positive real shock, the real interest rate should decrease initially.<sup>5</sup> The latter, however, depends on how monetary policy responds to current inflation. Increasing the response to inflation through a larger  $\Phi_\pi$  (and/or smaller  $\rho_i$  and

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<sup>5</sup>This is what Steinsson (2008) argues to be the necessary condition for the real exchange rate to exhibit the desired behaviour. It also implies that only shocks that cause negative co-variation between output and prices can give rise to hump-shaped dynamics of the real exchange rate.

$\Phi_y$ ) tends to decrease the degree of hump in the impulse responses. Table 2 proves the latter assertion. It shows that an increase in  $\Phi_\pi$  results in a decrease in  $\frac{UP}{HL}$  and  $2HL - QL$ .

$\Phi_\pi$	$HL$	$2HL - QL$	$\frac{UP}{HL}$	$Corr(\Delta e, \Delta q)$
<b>2.8</b>	2.95	1.28	0.36	0.94
<b>2.4</b>	3.19	1.53	0.42	0.90
<b>Baseline</b>	3.61	1.95	0.50	0.77
<b>1.6</b>	4.54	2.89	0.62	0.33
<b>1.2</b>	9.23	7.59	0.82	-0.15

Notes: Half-life ( $HL$ ) measures the largest time  $T$  such that  $IR(T - 1) \geq 0.5$  and  $IR(T) < 0.5$ , where  $IR(T)$  the value of the impulse response function in period  $T$  from a unit-sized impulse in period 0. Up-life ( $UP$ ) is the largest  $T$  such that  $IR(T) \geq 1$  and  $IR(T) < 1$ . Quarter-life ( $QL$ ) is the largest  $T$  such that  $IR(T - 1) \geq 0.25$  and  $IR(T) < 0.25$ .  $HL$ ,  $QL$  and  $UP$  are measured in years.

**Table 2: Real exchange rate dynamics for different degrees of interest rate smoothing**

Table 2 reveals another interesting feature of the model. While there is a negative link between the strength to inflation response and the degree of hump in the real exchange rate dynamics, the correlation between the nominal and the real currency returns rises as monetary policy becomes more responsive to inflation developments. This actually implies that the baseline framework is not fully capable to simultaneously rationalize the hump-shaped dynamics of the real exchange rate and the close co-variation between the nominal and real exchange rates across the adjustment process. Although the baseline model generates the observed degree of hump in the exchange rate response, it is not able to match the close co-movement between the nominal and real currency returns: the coefficient of correlation is only 0.77, way below from what can be observed in data.<sup>6</sup> Moreover, in the case of  $\Phi_\pi = 1.2$ , there is a negative co-movement between the nominal and real currency returns.

The positive link between the inflation response and the co-movement between the real and nominal returns is not specific to the particular calibration for the non-policy parameters. The next section takes a closer look at this matter by considering the analytical solution of

<sup>6</sup>The correlation between nominal and real exchange rate returns is close to unity in data (see, e.g. Monacelli (2004)).

a simplified version of the baseline model.

## 4. Co-movement between the nominal and the real exchange rate: The role of monetary policy

Assume that  $\rho_i = 0$ . While interest-rate smoothing is crucial in generating persistent dynamics of the real exchange rate, it only affects the co-movement between the nominal and real currency returns by decreasing the strength of policy response to inflation. Therefore, without loss of generality, one can analyze the co-movement between the nominal and real exchange returns only by considering different calibrations for policy response parameters,  $\Phi_\pi$  and  $\Phi_y$ .

Using the method of undetermined coefficients, one can get the following policy functions for domestic inflation and output:

$$\pi_{ht} = \beta_u u_t \tag{4.1}$$

$$y_t = \alpha_u u_t \tag{4.2}$$

where  $\beta_u = \frac{\sigma(1-\rho_u)+\Phi_y}{(1-\beta\rho_u)(\sigma-\sigma\rho_u+\Phi_y)+\kappa(\Phi_\pi-\rho_u)} > 0$  and  $\alpha_u = -\frac{\Phi_\pi-\rho_u}{(1-\beta\rho_u)(\sigma-\sigma\rho_u+\Phi_y)+\kappa(\Phi_\pi-\rho_u)} < 0$

The policy functions for the currency returns can be recovered by using the equilibrium conditions of the model. In particular, one can get:

$$\Delta q_t = (1 - \alpha)\sigma\alpha_u\Delta u_t \tag{4.3}$$

$$\Delta e_t = \sigma\alpha_u\Delta u_t + \beta_u u_t \tag{4.4}$$

Consider the impact of an increase in a supply shock on nominal and real currency returns.

We have:

$$\frac{\partial\Delta q_t}{\partial u_t} = (1 - \alpha)\sigma\alpha_u = -\frac{(1 - \alpha)\sigma\Phi_\pi}{\kappa\Phi_\pi + \Phi_y + \sigma} < 0$$

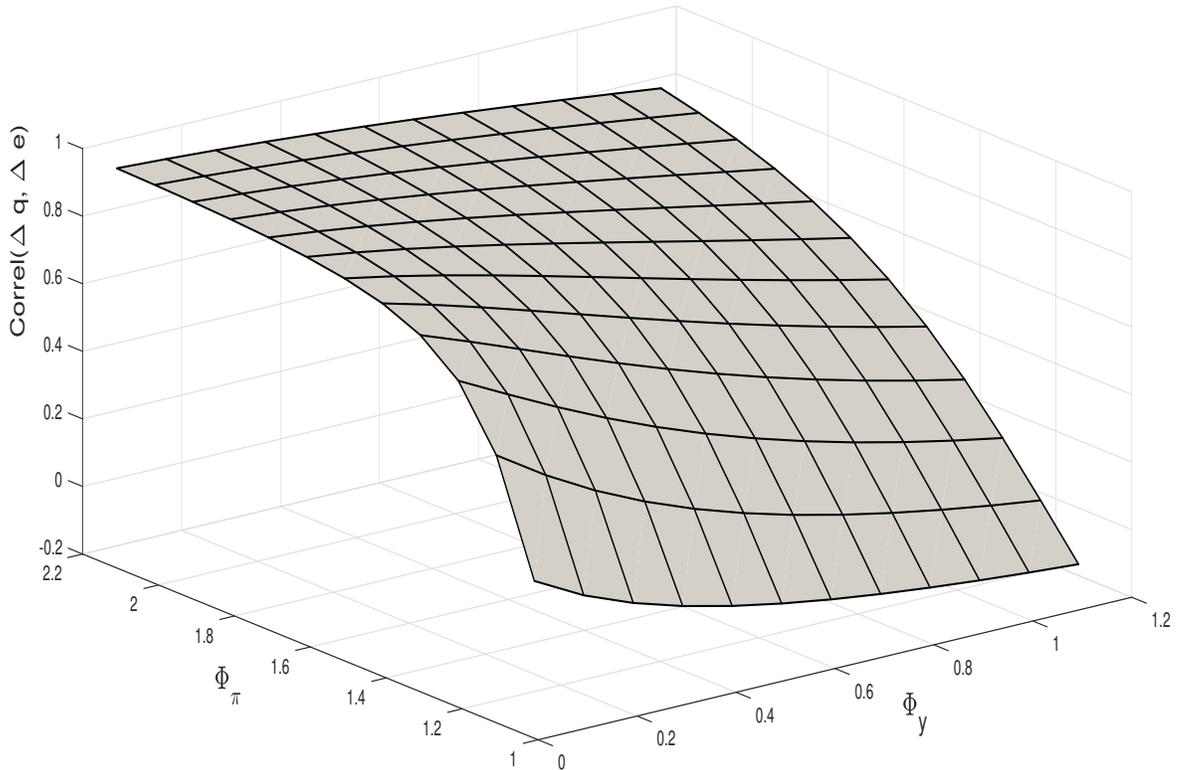
for all plausible calibrations. On the other hand:

$$\frac{\partial \Delta e_t}{\partial u_t} = \sigma a_u + \beta_u = \frac{\sigma(1 - \Phi_\pi) + \Phi_y}{(1 - \beta\rho_u)(\sigma - \sigma\rho_u + \Phi_y) + \kappa(\Phi_\pi - \rho_u)} \leq 0$$

i.e. the effect of a supply shock on the nominal return, on the contrary, depends upon the parameters of the Taylor rule. In particular, the nominal exchange rate depreciates following an increase in real shocks if there is a moderate response to inflation and a strong response to output. The logic for this result goes as follows. As a response to an increase in supply-side shocks, the policy rule calls for an increase in the interest rate. However, a Taylor rule with a non-zero response to output moves the interest rate less aggressively. Consequently, there is a substantial increase in domestic prices. PPP holds in the long run of the model. The latter exerts depreciation pressure on the currency (captured by  $\beta_u$ ). On the other hand, the increase in the nominal interest rate tends to appreciate the nominal exchange rate through the risk sharing channel (captured by  $a_u$ ). A weak response to inflation and/or a substantial response to output causes the PPP channel to outweigh the risk sharing channel. This result is consistent with that of Clarida and Waldman (2007). Consequently, the correlation between the real and the nominal returns decreases as the response to inflation becomes stronger and/or the response to output becomes weaker. Figure 2 confirms the latter. It plots the coefficient of correlation between the real and the nominal exchange rate for different calibrations of  $\Phi_\pi$  and  $\Phi_y$ .<sup>7</sup> We observe that the intuition from the baseline model is preserved here. A moderate response to inflation results in a low degree of correlation between the nominal and real currency returns. Moreover, for small values of  $\Phi_\pi$ , the coefficient of correlation becomes negative.

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<sup>7</sup>In all considered cases, the model has a unique equilibrium.



Notes: X-axis and Y-axis show the values of policy rule parameters. Z-axis shows the corresponding correlation coefficients between the real and the nominal currency returns.

**Figure 2. Correlation between nominal end real currency returns: The role of policy response parameters**

## 5. Conclusion

In the current paper, I show that the ability of the baseline small open economy model to replicate the actual behaviour of the real exchange rate crucially depends on the design of the monetary policy rule. In particular, a policy with a sluggish interest rate response to inflation is the key. Meanwhile, I also show that a moderate response to inflation implies a rather weak co-movement between the nominal and real currency returns. The latter stands in sharp contrast with empirical observations. In sum, the baseline model is not capable of fully capturing the real exchange rate adjustment process.

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